习题1

截止时间：10月15日

习题1. Let $E \subset \mathbb{R}^d$ be a bounded set, and $F \subset \mathbb{R}^d$ be an elementary set. Show that $m^{*,(J)}(E) = m^{*,(J)}(E \cap F) + m^{*,(J)}(E \backslash F)$.

习题2. Let $E \subset \mathbb{R}^d$ be Lebesgue measurable. Show that

$$m(E) = \sup_{K \subset E,\ K \text{ compact}} m(K).$$

习题3 (Inner measure)**.** Let $E \subset \mathbb{R}^d$ be a bounded set. Define the *Lebesgue inner measure* $m_*(E)$ of E by the formula

$$m_*(E) := m(A) - m^*(A \backslash E)$$

for any elementary set A containing E.

(1) Show that this definition is well defined, i.e., that if A, A' are two elementary sets containing E, that $m(A) - m^*(A \backslash E)$ is equal to $m(A') - m^*(A' \backslash E)$.

(2) Show that $m_*(E) \leq m^*(E)$, and that equality holds if and only if E is Lebesgue measurable.

习题4. 课本P24 第5题