

第 8 次作业

截止日期：4 月 21 日

习题 1. 课本 P112 练习 3.5(2).

解. $(t-x; t]; [t, t+x]$. □

习题 2. 课本 P118 练习 3.6(3).

解. 我们有

$$P(X \leq x) = \begin{cases} \frac{x}{b} & 0 \leq x \leq b \\ 1 & x > b. \end{cases}$$
$$P(Y \leq x) = \begin{cases} \frac{x}{b}(2 - \frac{x}{b}) & 0 \leq x \leq b \\ 1 & x > b. \end{cases}$$

注意到当 $0 \leq x \leq b$ 时, $2 - \frac{x}{b} \geq 1$, 所以 $P(X \leq x) \leq P(Y \leq x)$, 于是 X 随机大于 Y . □

习题 3. 课本 P133 习题 3.13.

证明. 当 t 充分大时, 剩余寿命分布函数近似为 $\frac{1}{\mu} \int_0^t \bar{F}(s) ds$, 于是密度函数近似为

$$\frac{1}{\mu} \bar{F}(y) = (\lambda/k!) \int_{\lambda y}^{\infty} s^{k-1} e^{-s} ds. \quad \square$$

习题 4. 课本 P133 习题 3.15.

解. 我们有

$$\begin{aligned} P(R(t) > x | A(t) = s, N(t) = n) &= P(S_{n+1} - t > x | t - S_n = s, S_{n+1} > t) \\ &= \frac{P(S_{n+1} - t > x, t - S_n = s, S_{n+1} > t)}{P(t - S_n = s, S_{n+1} > t)} \\ &= \frac{P(X_{n+1} > x + s, S_n = t - s)}{P(X_{n+1} > s, S_n = t - s)} \\ &= \frac{P(X_{n+1} > x + s)}{P(X_{n+1} > s)} = \frac{\bar{F}(x + s)}{\bar{F}(s)}, \end{aligned}$$

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于是

$$\begin{aligned}
 P(R(t) > x | A(t) = s) &= \sum_{n=0}^{\infty} P(R(t) > x | A(t) = s, N(t) = n) P(N(t) = n | A(t) = s) \\
 &= \frac{\bar{F}(x+s)}{\bar{F}(s)} \sum_0^{\infty} P(N(t) = n | A(t) = s) \\
 &= \frac{\bar{F}(x+s)}{\bar{F}(s)}, s \leq t.
 \end{aligned}$$

类似地可得

$$P(R(t) > 2x | A(t+x) = s) = \frac{\bar{F}(s+x)}{\bar{F}(s)}, \quad s \geq x. \quad \square$$